I International Young Naturalists' Tournament

Republic of Belarus

Gymnasium №29

Team "Universum"

Problem #3

The magnetic arrows

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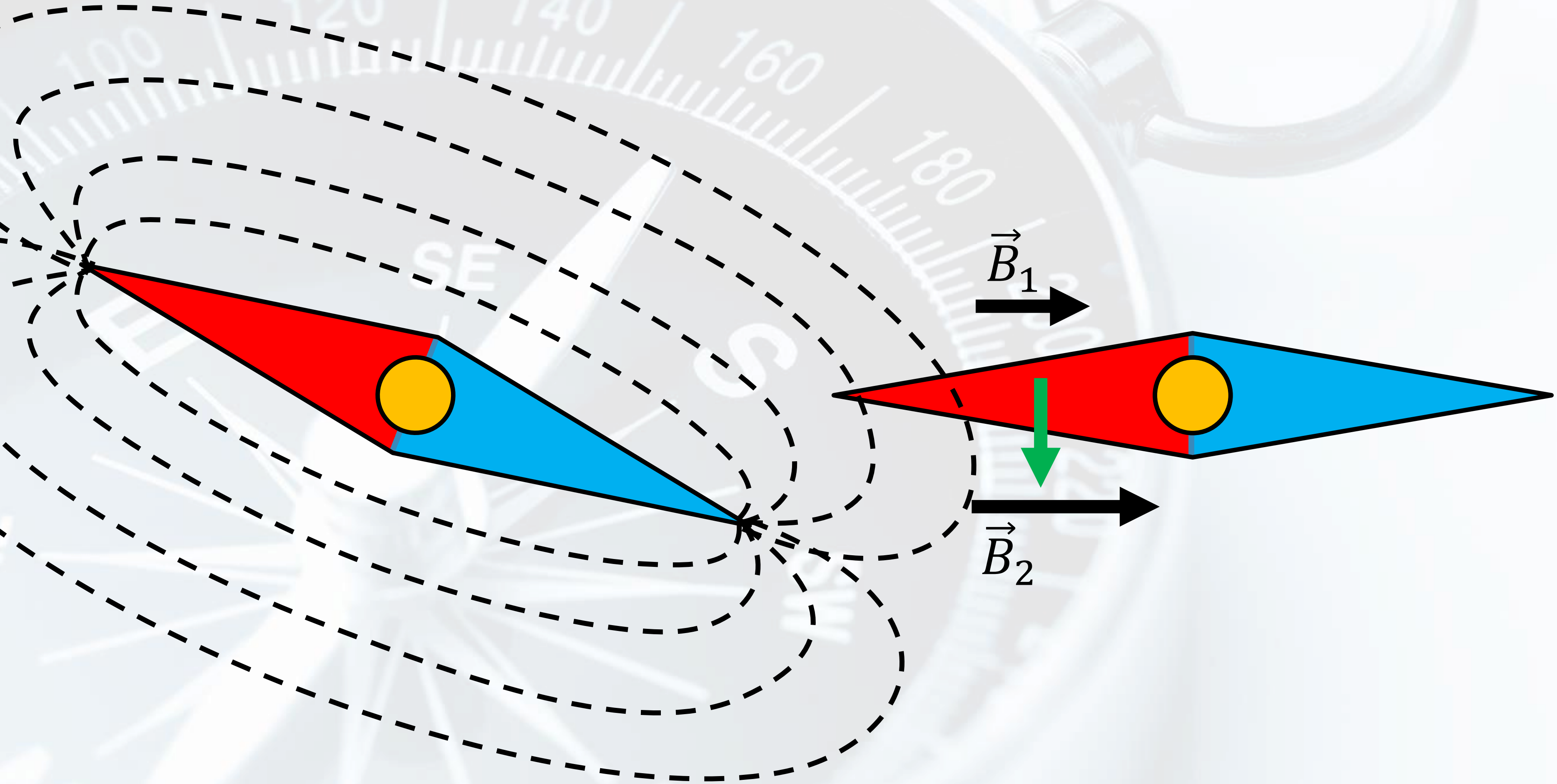
Formulation of the problem

Place two suspended magnetic arrows **close to each other**. After a short time they will reach the equilibrium where the opposite poles are aligned together. Deflect one of the arrows by some **small angle** and release it. Both arrows will start to oscillate. **Investigate** and **explain** the character of the **coupled oscillations** of the magnetic arrows

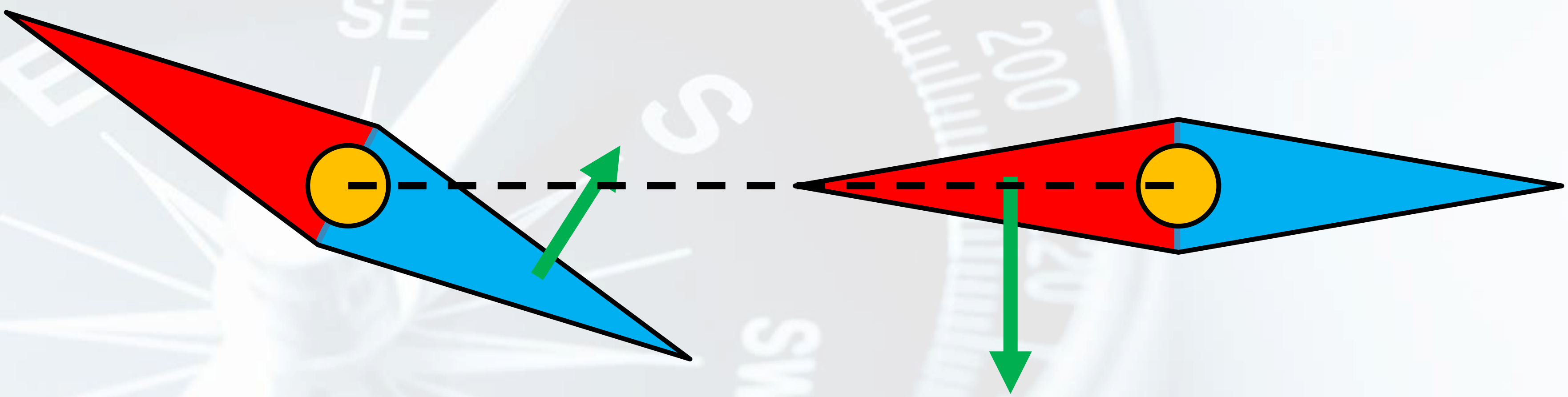
Plan of our work

- Qualitative explanation
- Investigation of the effect
- Theoretical model
- Conclusions and results

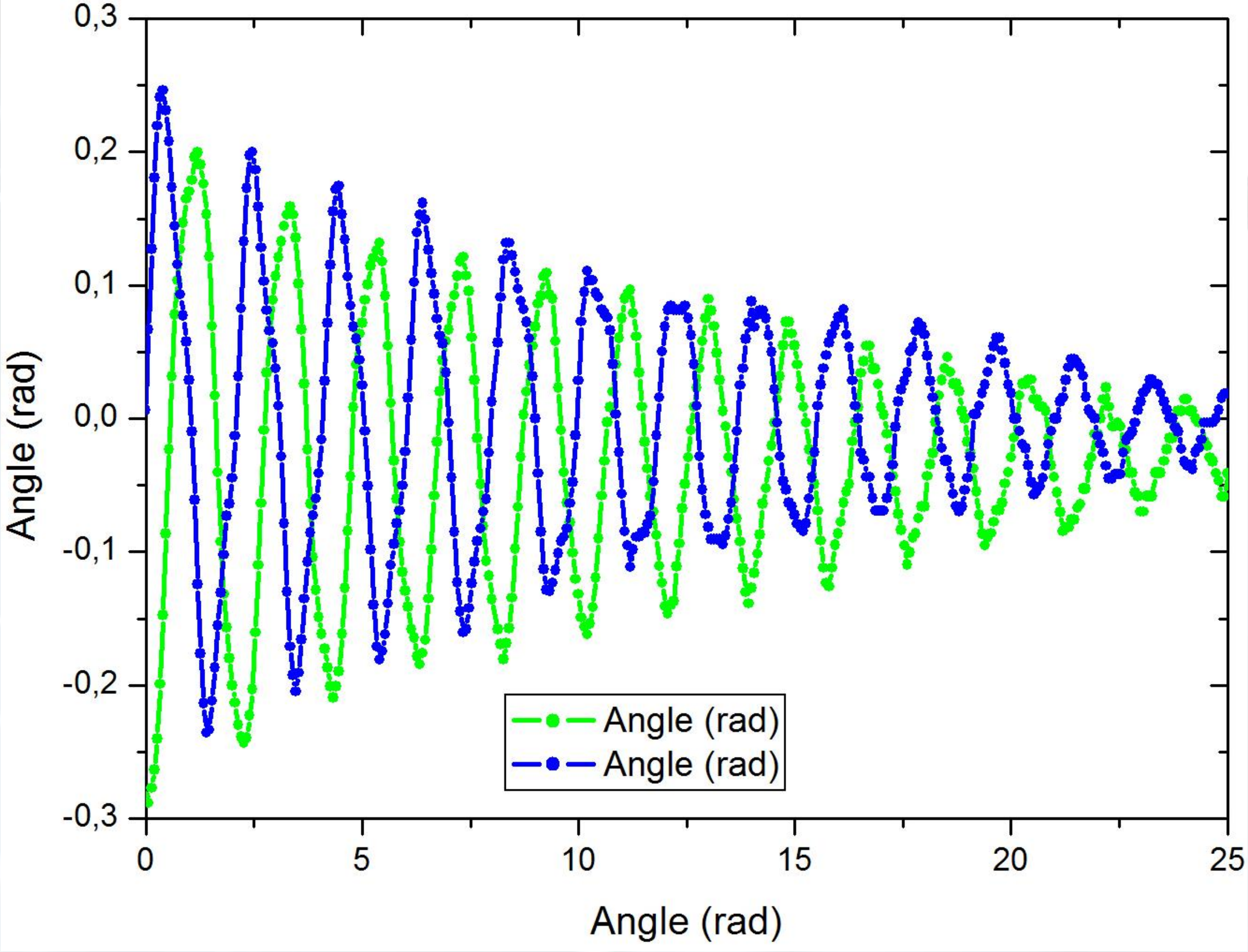
Qualitative explanation

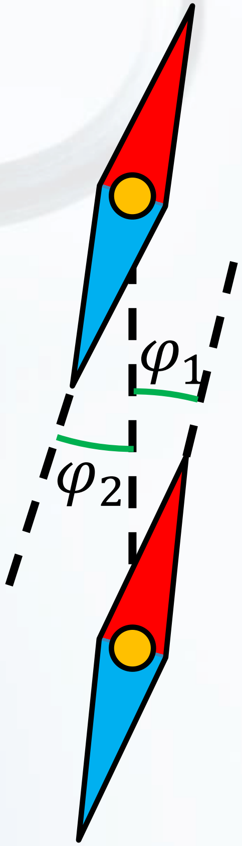


Appearance of oscillations

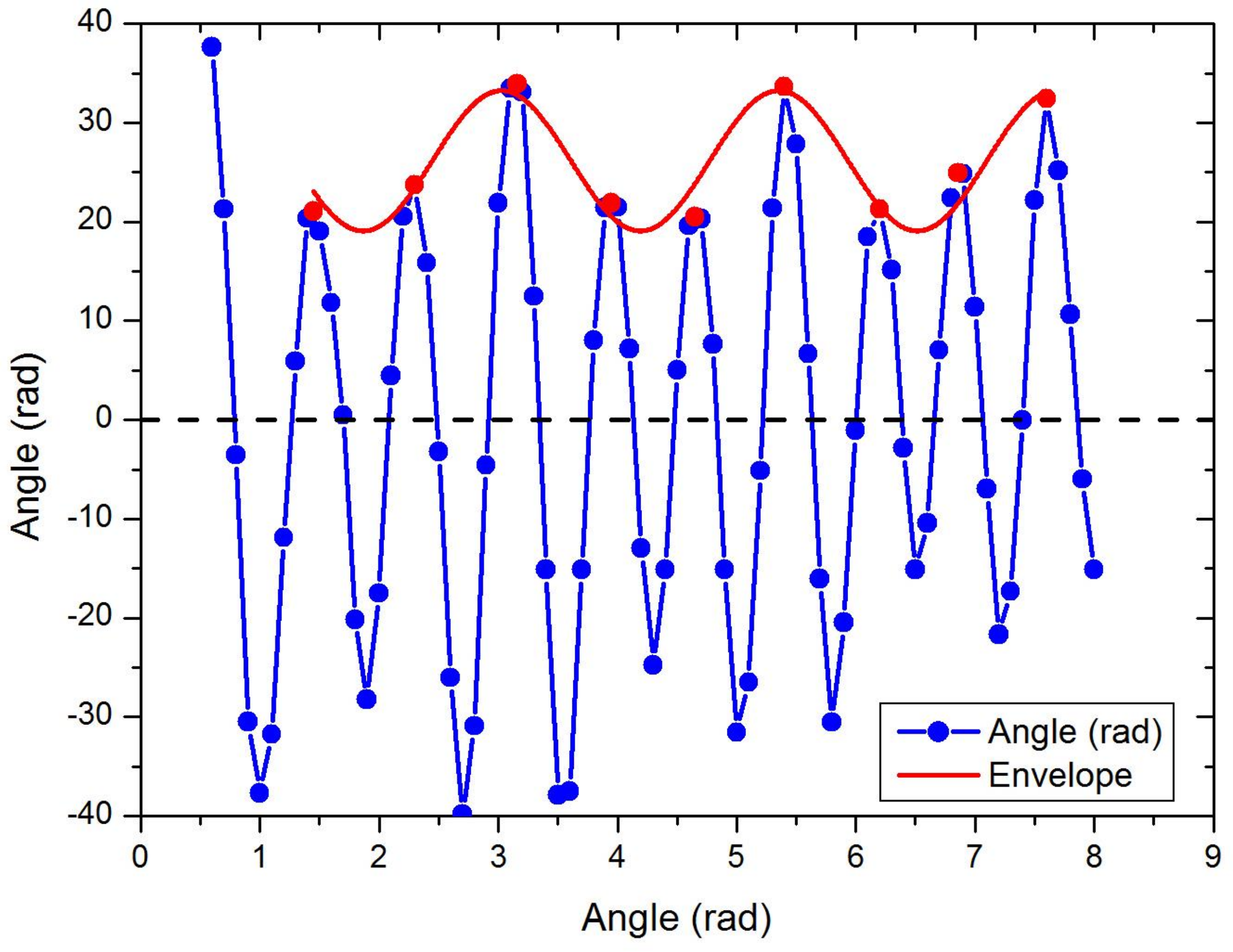


Dependence of angle on time



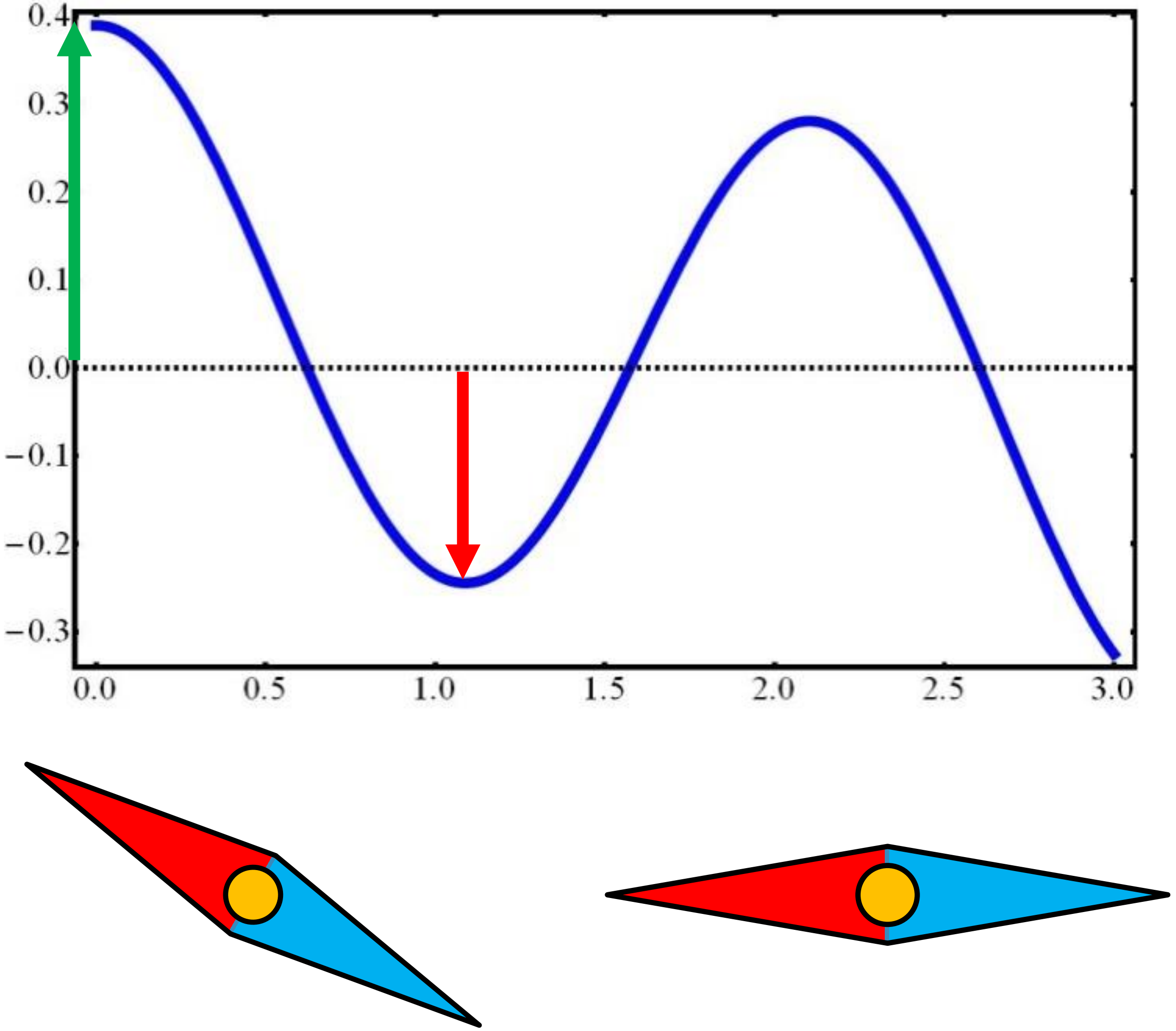


Sum of oscillations



$$A_1 \cos(\omega_1 t + \varphi_1) + A_2 \cos(\omega_2 t + \varphi_2)$$

Sum of oscillations



Different modes of oscillations

$$\boxed{A_1\cos(\omega_1 t+\varphi_1)+A_2\cos(\omega_2 t+\varphi_2)}$$

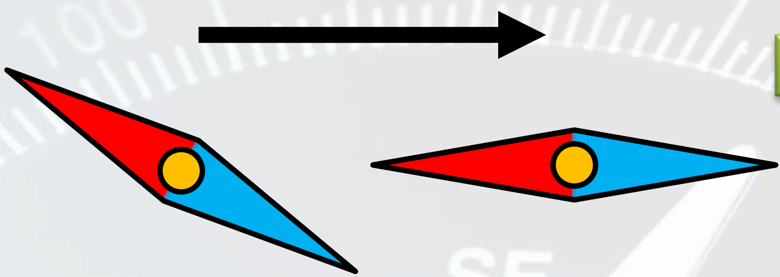
Mixed-phase (non-harmonic) oscillations

Anti-phase (harmonic) oscillations

In-phase (harmonic) oscillations

Influence of distance

$\vec{B}_E$

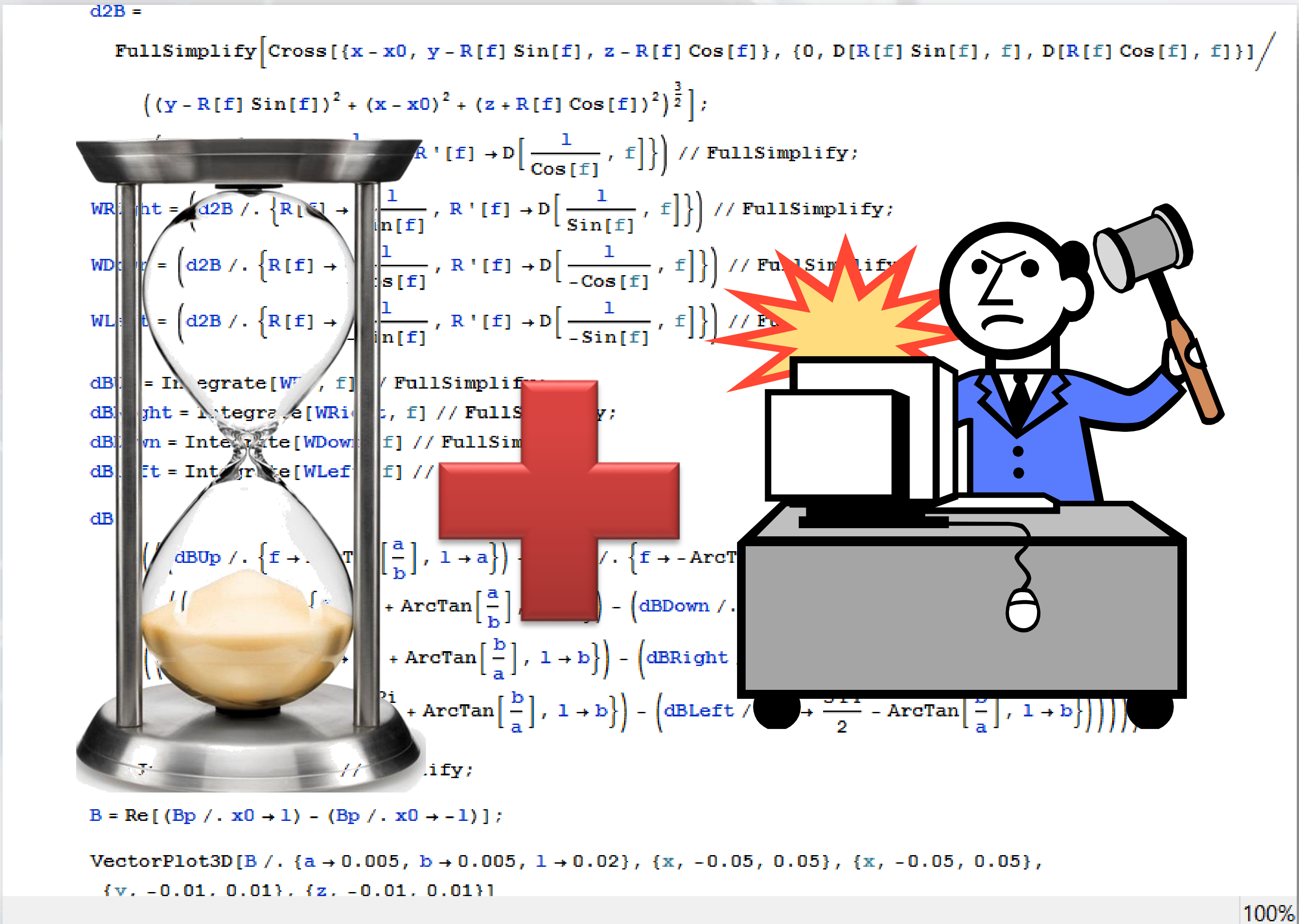


$$\vec{F} = \underbrace{\vec{F}_E}_{\boxed{\text{const}}} + \underbrace{\vec{F}(D,\varphi)}_{\text{decreasing with distance}}$$

$B_a \gg B_E$	$B_a \approx B_E$	$B_a \ll B_E$ D
Normal oscillations	Slower oscillations with influence of Earth magnetic field	Oscillations of a solitary arrow

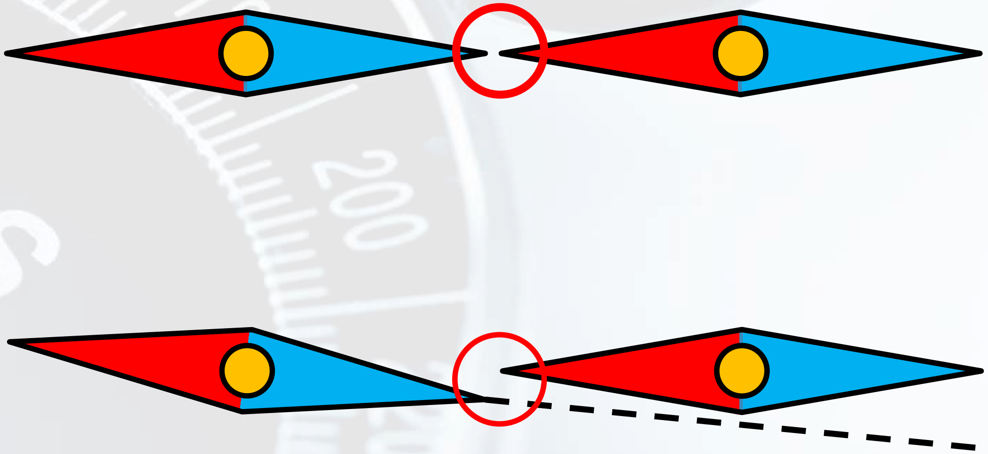
Place two suspended magnetic arrows **close to each other**….

Full theoretical model



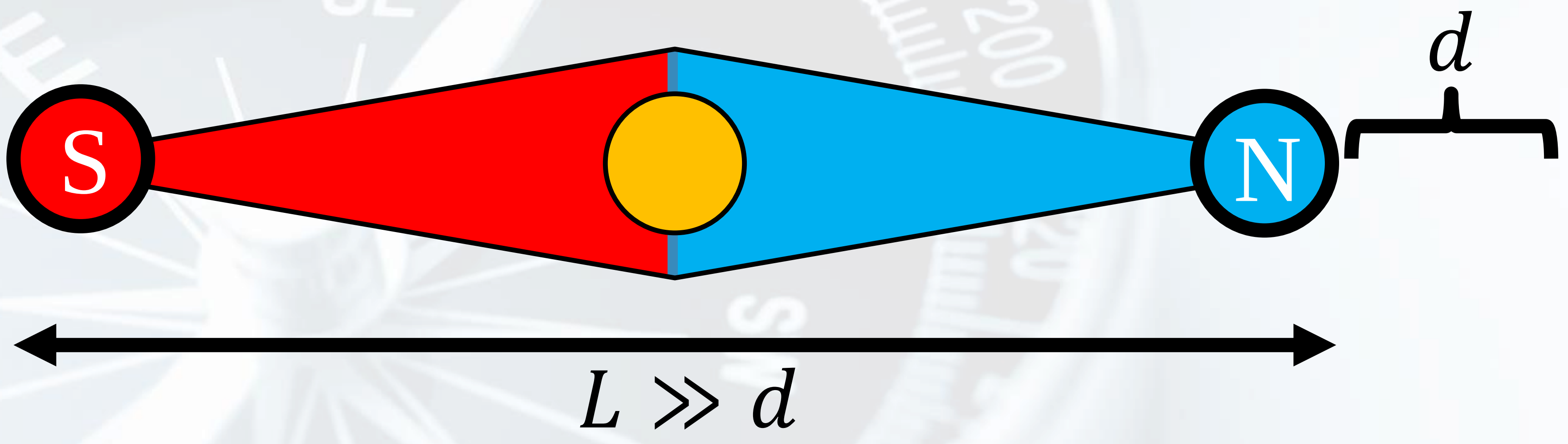
Main principles

- ✓ The distance between the ends of the arrows is very small
- ✓ The angles of deflection are very small

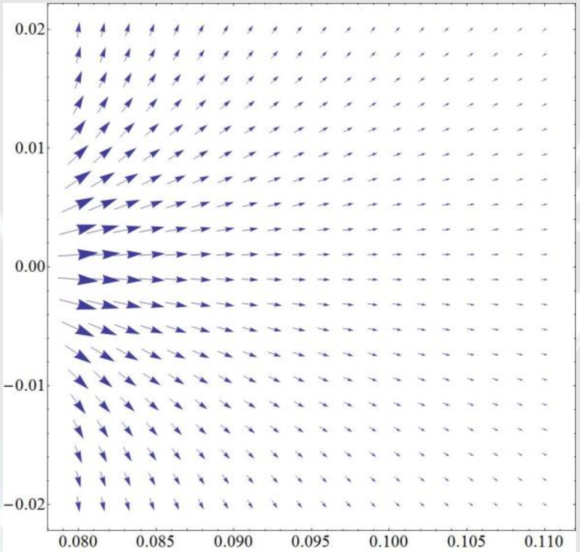


Close to each other, small angle

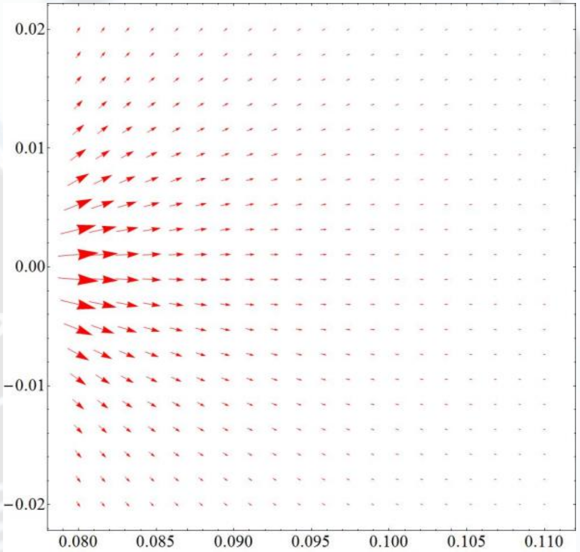
Magnetic field of the arrow



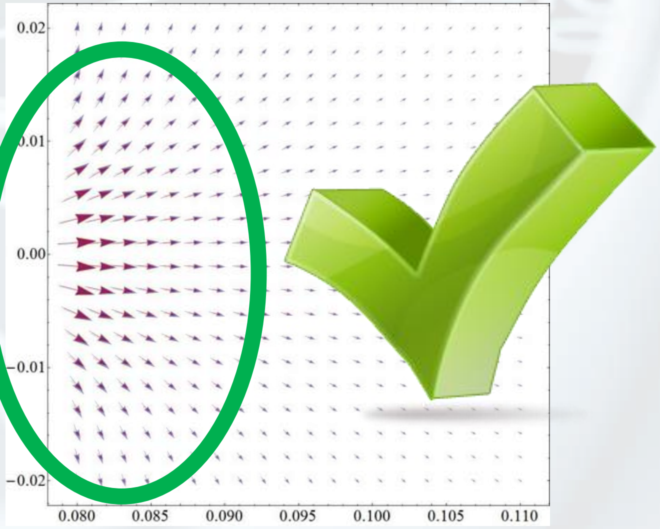
Magnetic field of the arrow



Honestly calculated

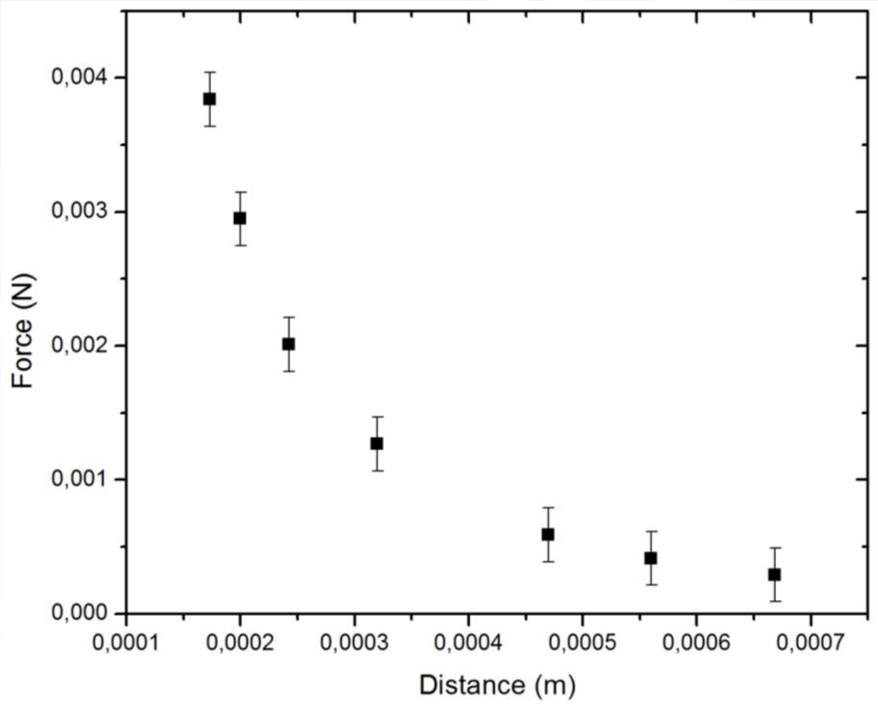


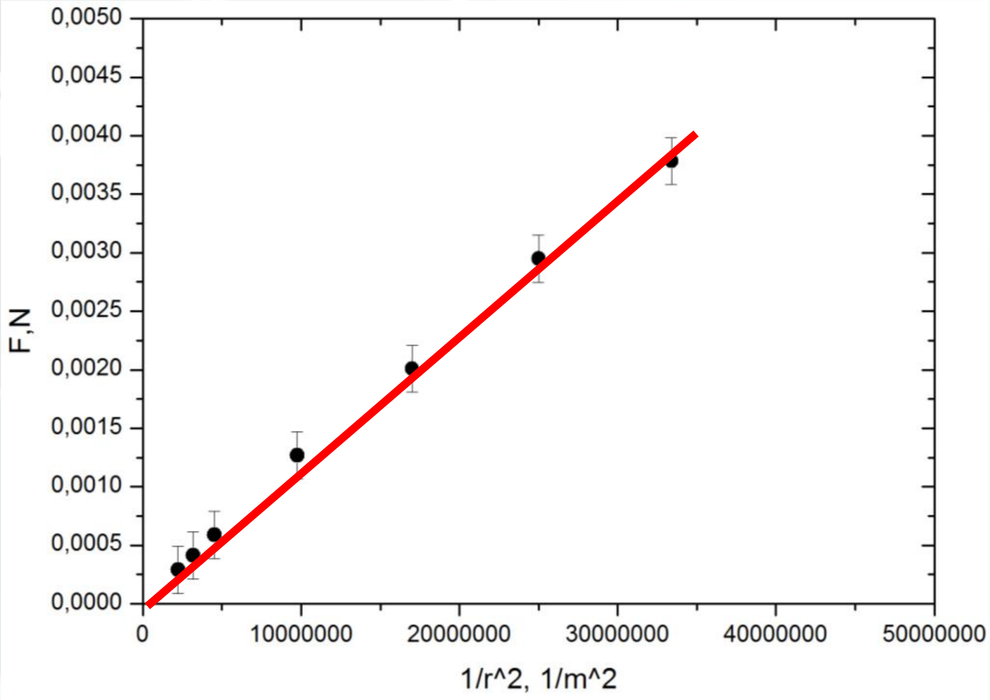
Our approximation



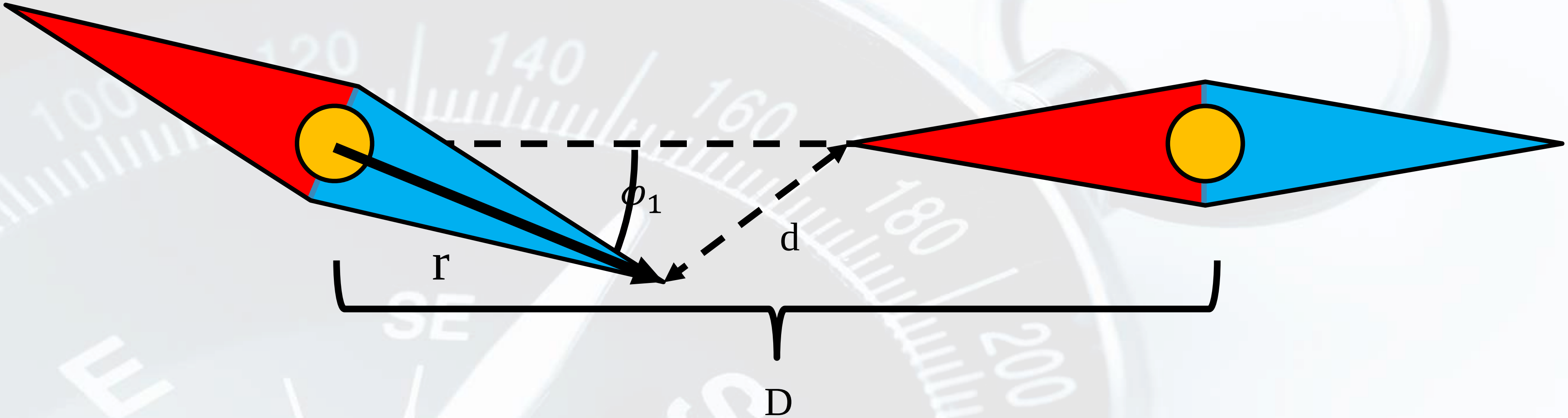
Attraction force

$$F \sim \frac{1}{r^2}$$





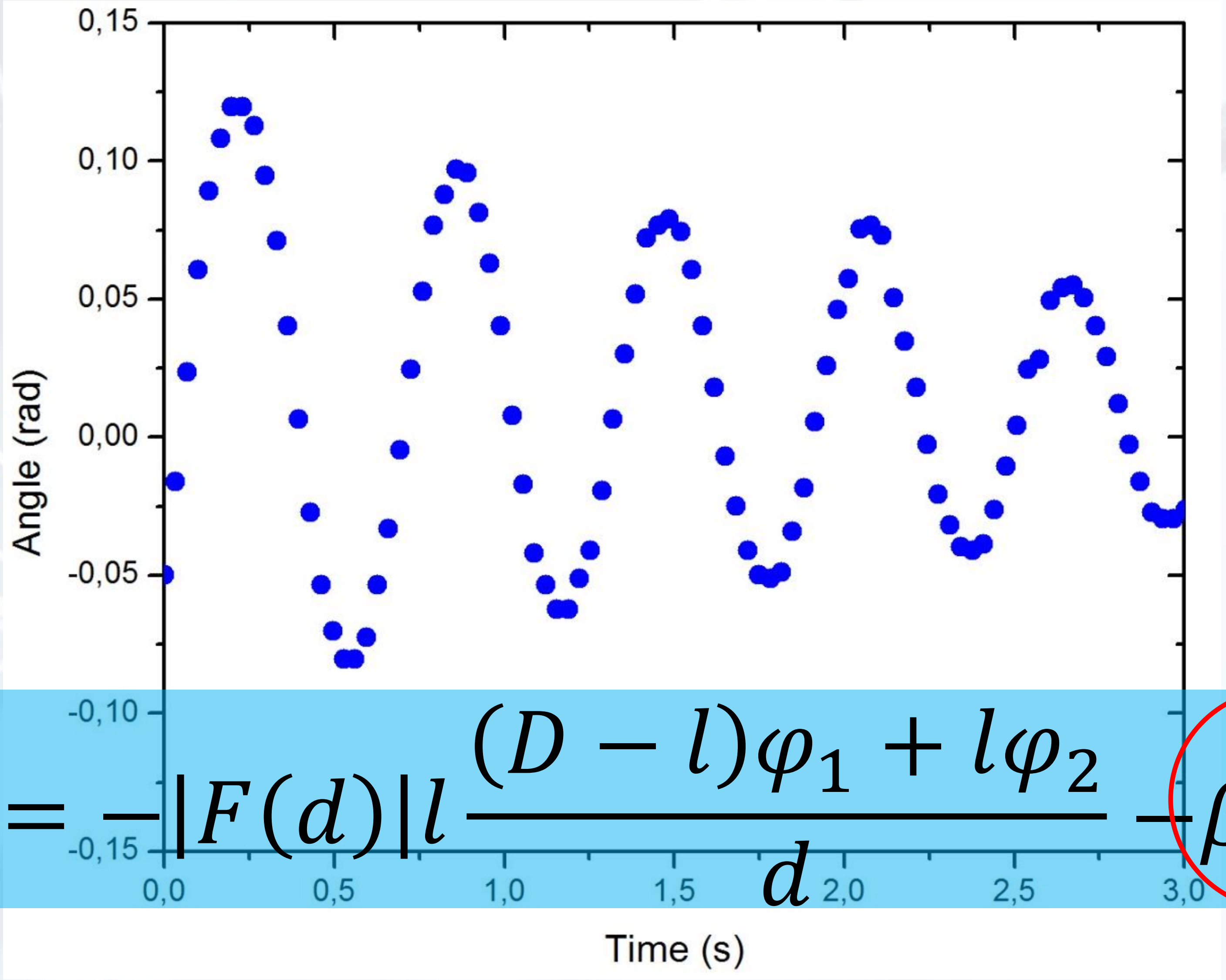
Law of motion



$$I\ddot{\varphi}_1 = M(\varphi_1, \varphi_2, d) = (\vec{r} \times \vec{F}(\vec{d}))_z$$

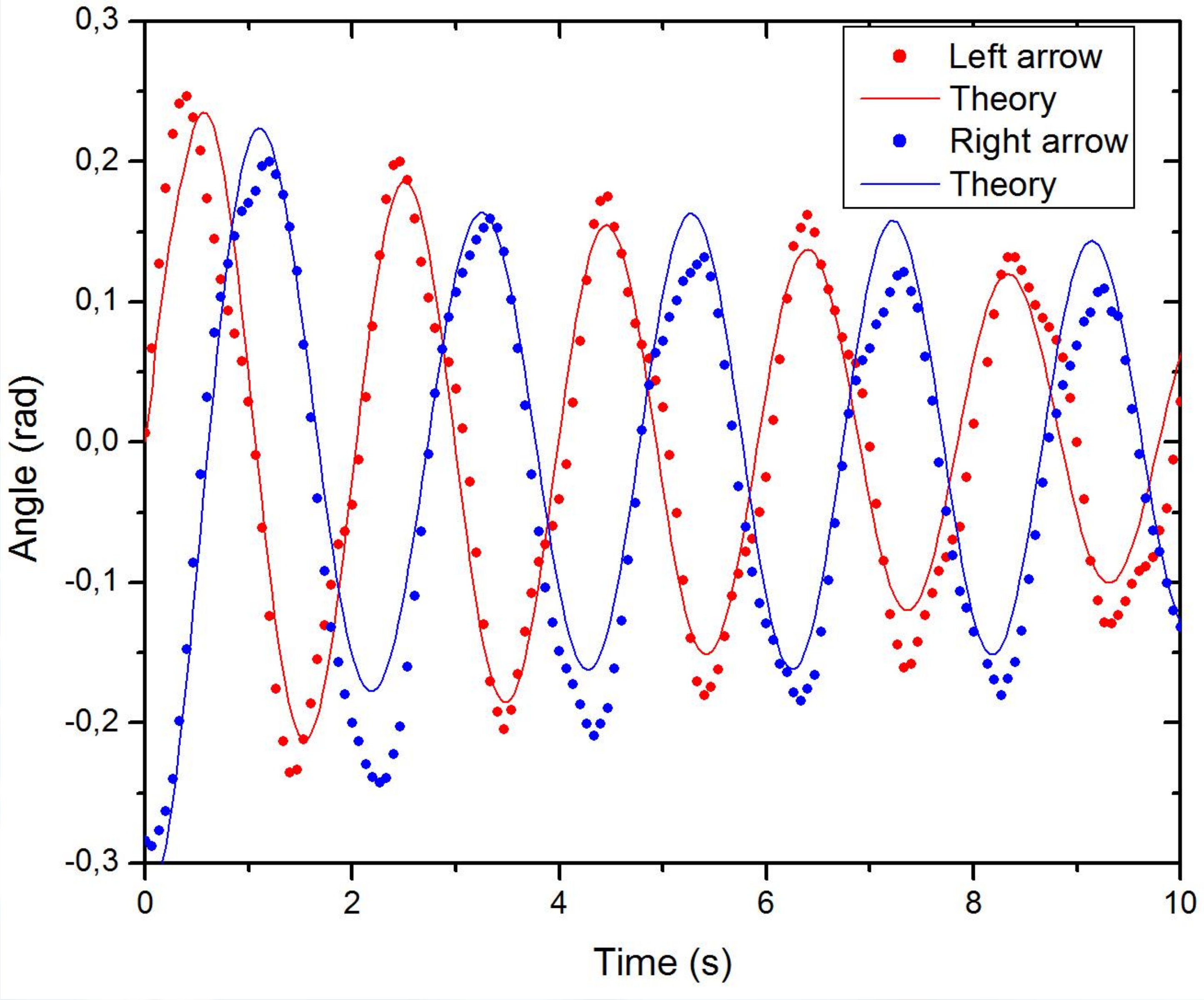
$$I\ddot{\varphi}_1 = -|F(d)|l\frac{(D-l)\varphi_1 + l\varphi_2}{d}$$

Damping of oscillations



$$I\ddot{\varphi}_1 = -|F(d)|l\,\frac{(D-l)\varphi_1 + l\varphi_2}{d} - \beta\dot{\varphi}_1$$

Comparison



Left arrow
Theory
Right arrow
Theory
Angle (rad)
Time (s)
0,3
0,2
0,1
0,0
-0,1
-0,2
-0,3
0
2
4
6
8
10

Results

Explain the phenomenon

- The presence of attractive force
- Appearing of oscillations

Investigate the phenomenon

- Form of oscillations
- Different modes of oscillations

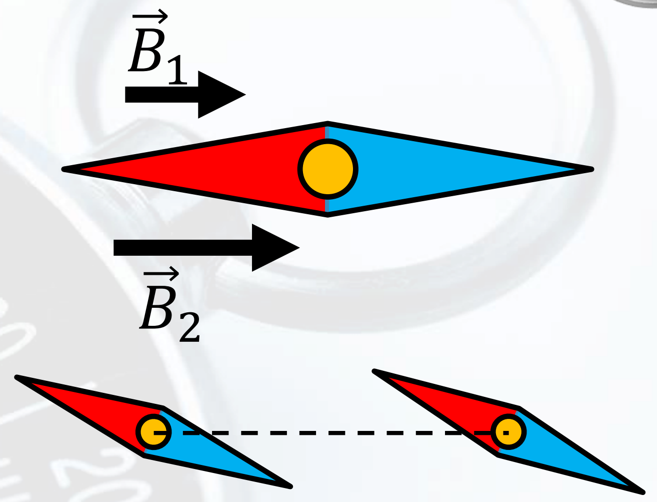
Theoretical model

- Good convergence with experiments

Conclusions

The explanation of the effect

- The attractive force and the rotating torque as a result appears because of inhomogeneity 0f the magnetic field
- The oscillations appear because the system has only one stable equilibrium position



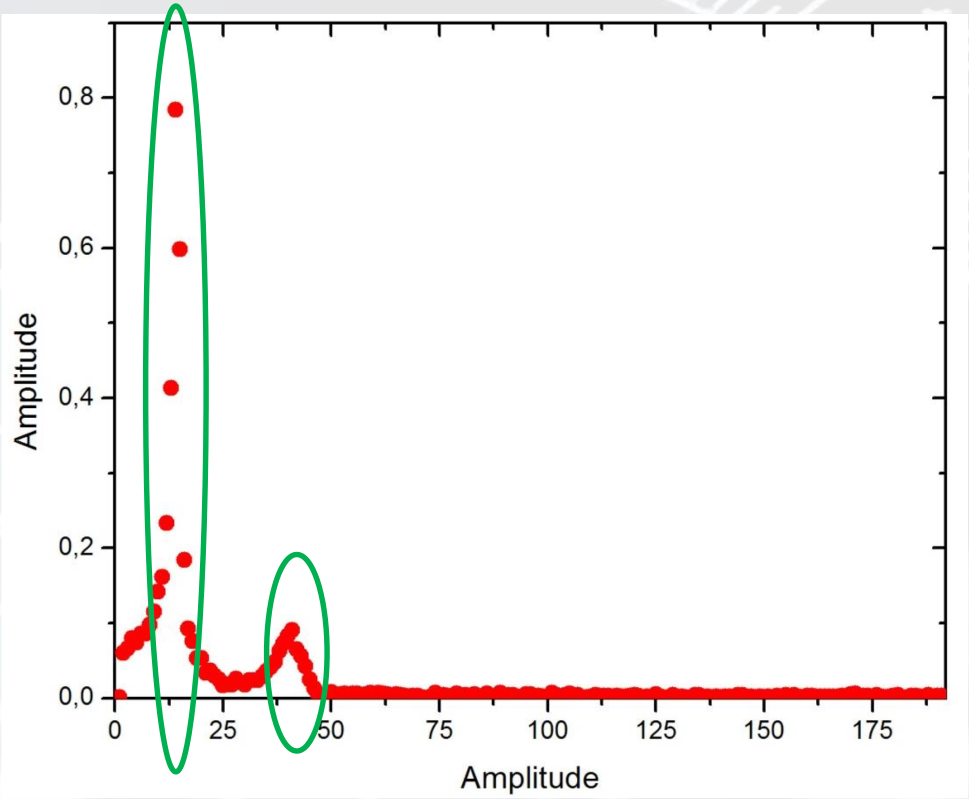
The results of investigation

- Oscillations represent a sum of two oscillations
- The difference of phase between this two oscillations define the exact form of oscillations
- The difference of phase of oscillations is defined by the initial conditions and the conditions of the experiment

$$\boxed{\vec{B}_{1,2} = f(D, l, \varphi_1, \varphi_2)}$$

Thank you for attention!

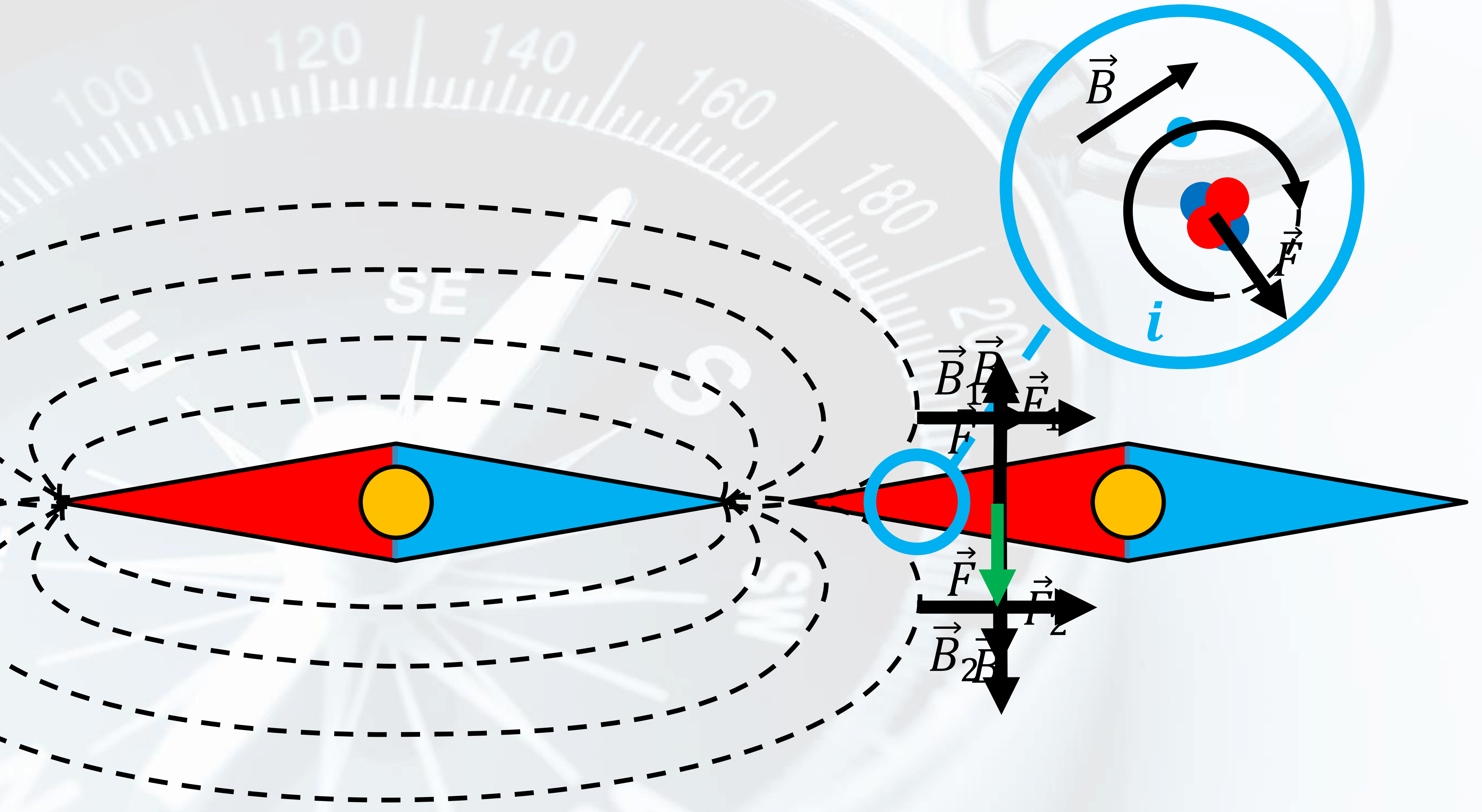
Extra slide: Fourier analysis for the oscillations



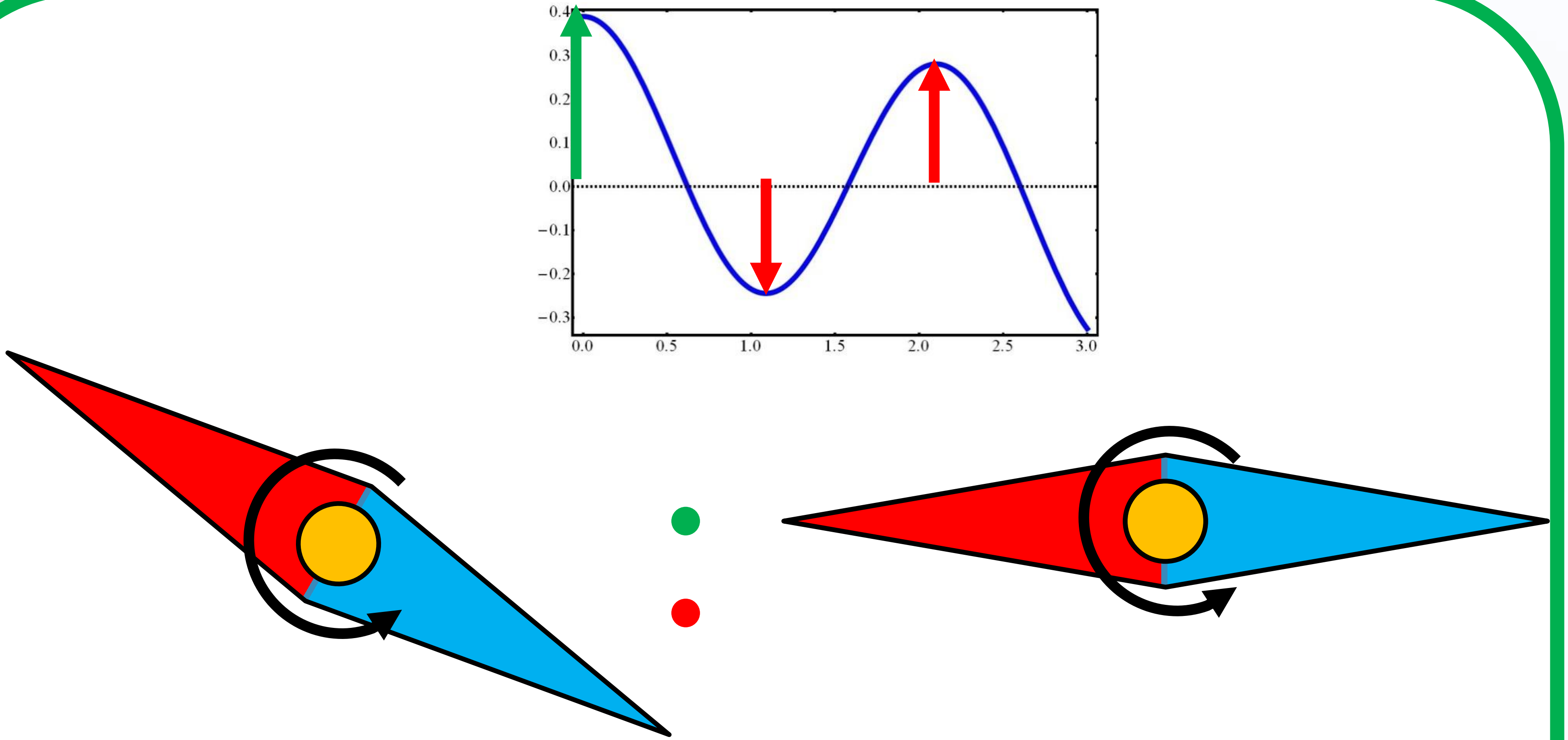
$$\omega_1 = \sqrt{\frac{D^5 I}{\alpha l}}$$

$$\omega_1 = \sqrt{\frac{D^4 (D - 2l) I}{\alpha l}}$$

Qualitative explanation

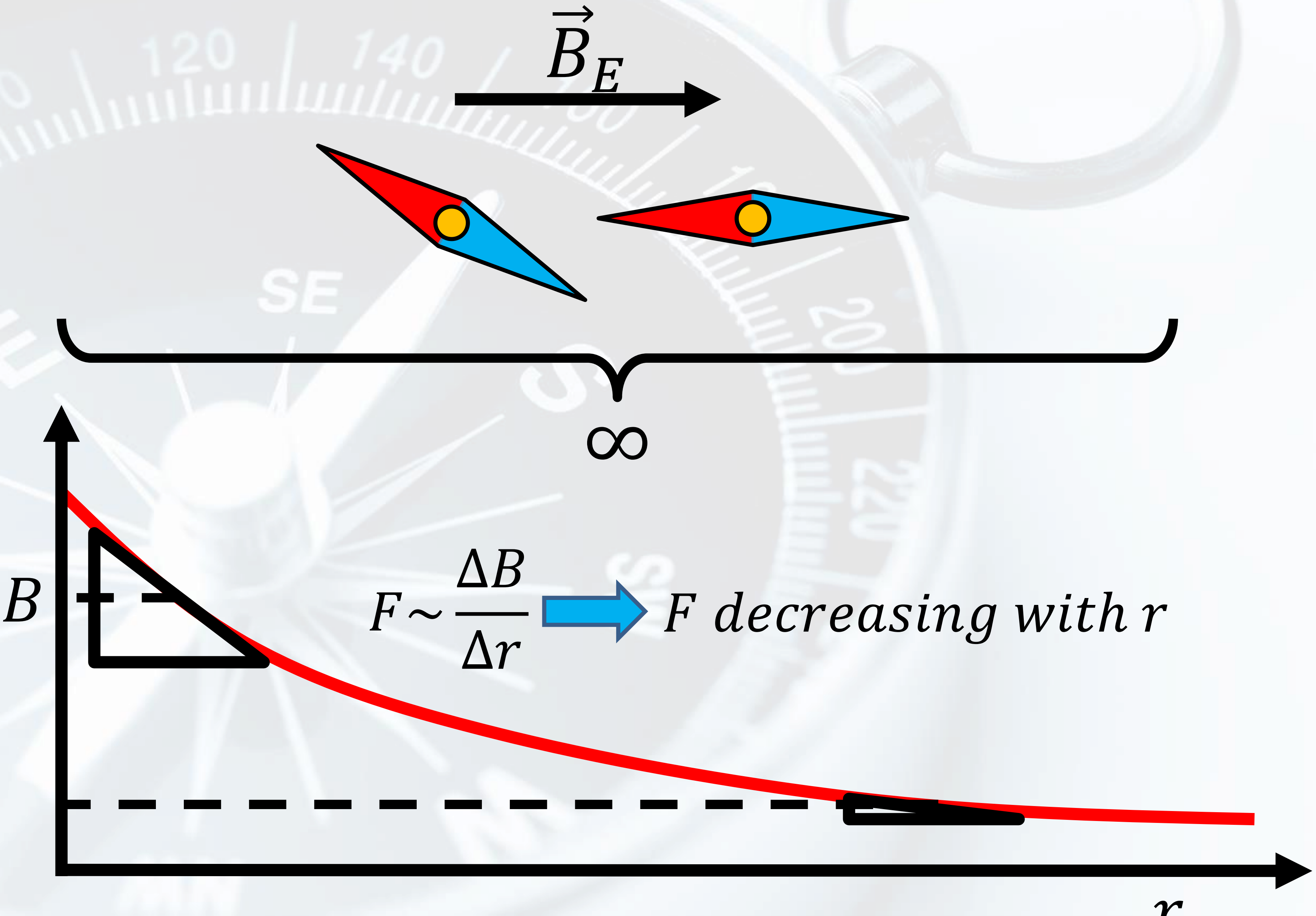


Sum of oscillations



$$A(t)\cos(\omega_0 t) = \mathrm{A}\cos(\Omega t + \Phi) + \mathrm{a}\cos(\omega t + \varphi)$$

Influence of distance



$F \sim \frac{\Delta B}{\Delta r}$ ⇒ F *decreasing with* r